Efficient real-time computation of harmonics in power systems using multiple reference frames

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***Abstract*—Harmonic extraction algorithms are widely used in the industry for electrical measurements. For real-time applications, algorithms such as the Fast Fourier Transform (FFT) and the Kalman Filter (KF) are frequently used. Unfortunately, FFTs impose certain restrictions to the sample time and yields inaccurate results in case of small frequency variations. KF overcomes some of these problems, but have a high computational cost and an unexpected behaviour if there is any un-modelled harmonics in the signal. This paper proposes a robust method to compute harmonic components in real-time with a moderate computational cost. The proposed method is based on Multiple Reference Frames rotating at each harmonic frequency. Once rotated, each harmonic is a DC magnitude in the corresponding rotating frame and can be extracted using a low-pass filter. The main contribution of this paper is the method proposed to transform the measured signals with a minimum use of trigonometric functions. Accordingly, the computation time is drastically reduced and the implementation on a DSP becomes very simply. The proposed algorithm, based on Moivre's theorem, is valid for both three- and single-phase signals. The algorithm has been tested on a 5 kVA prototype of a Series Active Power Filter. The results reveal that harmonics can be accurately estimated in a reasonable time with low computational cost.**

***Index Terms*—Power conversion harmonics, digital signal processing, power quality, power harmonic filters.**

I. Introduction

Electric power quality has attracted considerable attention in recent years, mainly due to the harmonic distortion generated by power electronics converters. This harmonic distortion deteriorates the quality of the main magnitudes of the grid, bringing on failures in equipments connected to the power system. To tackle this problem, appropriate standards have emerged to ensure proper operation and compatibility of all the equipment connected to the grid [1]. These standards impose limits on the amount of both current harmonics injected to the grid, and voltage harmonics present in the point of common coupling. To asses whether the objectives set by the regulations are met, appropriate processing of measurements is required [2].

For highly polluted grids, Passive and Active Power Filters are commonly used to reduce harmonic distortion [3]. Now-a-days, Active Power Filters are a booming technology, since these devices are capable of drastically reducing both current and voltage harmonics. These devices, although viable, need fast and reliable Digital Signal Processors (DSPs) to perform many calculations in real-time [4], including, digital processing of harmonics which bring a high computational cost. This fact has fostered research in harmonic estimation algorithms, with special emphasis on computational cost [2].

The Fast Fourier Transform (FFT) is commonly used in real-time applications to estimate the Discrete Fourier Transform (DFT) coefficients of a signal [5]. In fact, commercial DSPs has their own optimized routines to perform FFTs. However, FFTs suffer from several problems [6]. Firstly, the sample time is imposed by the frequency of the grid voltage, and with a small mismatch in the frequency, the FFT suffers from leakage. Secondly, random-frequency signals, like inter-harmonics, deteriorate the results. To overcome these problems, many modifications to the original FFT algorithm have emerge in recent years [7]-[8]. Even though some solutions yields reasonable results, they add computational burden [9]. The Kalman Filter (KF) has also become very popular in the harmonic-estimation field. Some versions of the KF can accurately estimate harmonics without linking the sample time to the grid period and without leakage problems. Unfortunately, the KF also has many drawbacks: high computational cost, unwanted behaviour in case of unexpected signals and an elaborated design process [10]. Other techniques, such as Artificial Neural Networks [11] have also been reported, but they are not very popular.

Synchronous Reference Frames (SRFs) are very popular in the field of power system analysis and power electronics [12]. Their application turns three-phase signals into space vectors, making the use of vector techniques very straightforward. Multiple Reference Frames (MRF) rotating at every harmonic frequency are often proposed in Active Power Filter control, but the implementations suggested are computationally very heavy [13] due to the abuse of trigonometric functions (sine and cosine). For this reason, MRFs are usually discarded as a method to calculate a large amount of harmonics.

This paper proposes a highly-efficient computation method to extract harmonic components from a signal using MRFs. Firstly, three-phase signals are transformed into vector form ($d-q$ axis). Signals are then referred to various reference frames rotating synchronously with each harmonic frequency where the corresponding harmonic can be extracted with a simple low-pass filter (LPF). The main a contribution of this paper is the method proposed to carry out these calculations in real-time with a minimum use of trigonometric functions. The required sine and cosine functions are computed using multiplications and sums by a suitable application of the Moivre's theorem [14]. As a consequence, the runtime of the algorithm is drastically reduced, easing its implementation on a low-cost DSP. Furthermore, the performance of the algorithm is independent of the sampling time and the grid frequency.

TABLE I
HARMONICS EQUIVALENCE BETWEEN abc AND $dq0$ REFERENCE FRAME

Harmonics in abc	abc sequence	Harmonics dq	Harmonics in 0
1	+	0	1
	0		
	-	2	
2	+	1	2
	0		
	-	3	
3	+	2	3
	0		
	-	4	
4	+	3	4
	0		
	-	5	
5	+	4	5
	0		
	-	6	
6	+	5	6
	0		
	-	7	
7	+	6	7
	0		
	-	8	
n	+	n-1	n
	0		
	-	n+1	

In this paper, the proposed algorithm is first presented with a recursive formulation for the three-phase case described above. Secondly, the proposed algorithm is also explained for single-phase signals. Finally, the proposed algorithm is tested on a 5 kVA prototype of a Series Active Power Filter, to asses the effectiveness of the filter.

II. HARMONICS AND SPACE VECTORS

Park's transform is widely used to study three-phase power systems. Using this transformation, one can calculate the so-called *dq0* components of the natural magnitudes of the three-phase system ($u_a(t)$, $u_b(t)$ and $u_c(t)$). The general expression for the Park's transform is:

$$u_{dq0}(t) = \mathbf{P}(\theta(t))u_{abc}(t) \tag{1}$$

with

$$\mathbf{P}(\theta) = k_2 \begin{bmatrix} k_1 & k_1 & k_1 \\ \cos(\theta) & \cos\left(\theta - \frac{2\pi}{3}\right) & \cos\left(\theta + \frac{2\pi}{3}\right) \\ -\sin(\theta) & -\sin\left(\theta - \frac{2\pi}{3}\right) & -\sin\left(\theta + \frac{2\pi}{3}\right) \end{bmatrix} \tag{2}$$

where subscript d stands for "direct", q stands for "quadrature" and 0 stands for "homopolar". The angle $\theta(t)$ can be chosen arbitrary, but it is often selected so that $d\theta(t)/dt$ is equal to the grid frequency and the d and q components of the grid voltage are DC magnitudes in an ideal situation (steady state and balanced with only positive sequence). It is then said that the natural variables have been referred to a reference frame synchronously rotating with angular speed $d\theta/dt$, which is considered the positive reference for angular speeds. Furthermore, $\theta(t)$ is typically chosen so that the q component of the grid voltage is zero. To this end, a Phase-Locked Loop (PLL) is typically used to calculate $\theta(t)$ [15]. For simplicity, the time dependence of signals will be omitted in the rest of the paper.

It can be easily proved that, using the Park's transform, only the positive sequence of the fundamental voltage is transformed into a constant value, whilst voltage harmonics and the negative sequence of the fundamental frequency of the grid voltage are moved to a different frequency. Table I summarizes the relationship between harmonics in abc (natural variables) and the harmonic number with respect to the grid frequency in $dq0$ variables (components). The so-called positive-sequence of a harmonic of frequency $n\omega$ in abc is transformed into a harmonic of frequency $(n-1)\omega$ in a dq reference frame ($n_{dq}^{+} = n_{abc}^{+} - 1$). In addition, the $d-q$ components of any of this harmonics can be put together in a space vector that rotates in the $d-q$ plane with positive speed $(n-1)\omega$. On the other hand, negative-sequence harmonics in abc of frequency $n\omega$ are transformed to harmonics of frequency $(n+1)\omega$ in a $d-q$ reference frame. ($n_{dq}^{-} = n_{abc}^{-} + 1$). In addition, the $d-q$ components of any of this harmonics can be put together in a space vector that rotates in the $d-q$ plane with negative speed $(n+1)\omega$. Homopolar harmonics do not modify their frequency ($n_0 = n_{abc}^{0}$). It is clear that if one wants to see harmonics as stationary vectors each of them must be referred to a reference frame rotating synchronously with its space vector and the angle used in (2) should be calculated as:

$$\theta_n^i = i \cdot n\theta \tag{3}$$

where n is the number of the harmonic in abc and i is 1 if the a positive-sequence element is considered and -1 if a negative-sequence component is considered. For example $n = 1$ and $i = 1$ hast to be used for the positive sequence of the fundamental component, $n = 1$ and $i = -1$ has to be used for the negative sequence of the fundamental component, $n = 2$ and $i = 1$ has to be used for the positive sequence of the second harmonic, and so on. It is also said that each reference frame has rotated an angle θ_n^i (i.e. $\theta_{n=1}^{i=1} = \theta$ for the positive-sequence of the fundamental harmonic and $\theta_{n=1}^{i=-1} = -\theta$ for the negative-sequence of the fundamental harmonic).

III. HARMONIC-EXTRACTION METHOD

A. Signals in a Synchronous Reference Frame

The dq components of a signal ($\boldsymbol{u}_{dq} = u_d + ju_q$) are obtained after referring the natural magnitude *abc* to a reference frame synchronously rotating with the grid voltage space vector. For this purpose the value of θ calculated by the PLL is used in (1). In order to refer the same signal to a reference frame synchronously rotating with the positive or negative sequence of a given harmonic n of the three-phase variable abc one has to do:

$$\boldsymbol{u}_{dq}^{n-i} = e^{-i \cdot j(n-i)\theta}\boldsymbol{u}_{dq} \tag{4}$$

because the angle $i(n-i)\theta$ is the angular difference between the positive sequence of the fundamental harmonic and any other harmonic of interest (n), positive sequence ($i = 1$) or negative negative ($i = -1$), which, in matrix form, is:

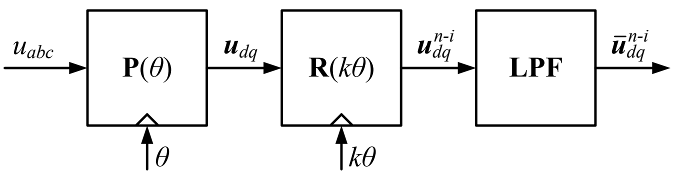

Fig. 1. Estimation of the n^{th} harmonic coefficients from signal u_{abc}.

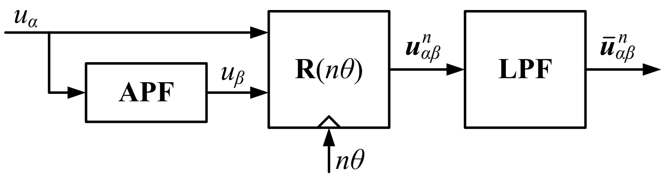

Fig. 2. Estimation of the n^{th} harmonic components from signal u_α.

$$\begin{bmatrix} u_d^{n-i} \\ u_q^{n-i} \end{bmatrix} = \underbrace{\begin{bmatrix} \cos k\theta & \sin k\theta \\ -\sin k\theta & \cos k\theta \end{bmatrix}}_{\mathbf{R}(k\theta)} \begin{bmatrix} u_d \\ u_q \end{bmatrix}; \quad k = i \cdot j(n-i) \tag{5}$$

Under this transformation, only the $d-q$ components of the $n-th$ harmonic of sequence i will be DC values whilst all the other components will be AC values of various frequencies. The DC values can be easily extracted using a LPF as shown in Fig. 1.

B. Homopolar component and single-phase signals

Generally speaking, Park's transform produces a space vector ($\boldsymbol{u}_{dq}$), and a scalar signal (u_0) for a three-phase variable. To be able to apply vector techniques to the homopolar component (or to a single-phase signal), it is necessary to artificially generate a quadrature component of the original signal [16]. If the single-phase (or homopolar) signal is:

$$u_\alpha[k] = A\cos(n\omega t_s k + \varphi) + e[k] \tag{6}$$

where t_s is the sampling time and $e[k]$ contains all other signals present in $u_\alpha[k]$. A signal in quadrature with (6) can be obtained with a digital all-pass filter (APF) with a phase delay of -90 degrees at frequency $n\omega$:

$$\frac{U_\beta(z)}{U_\alpha(z)} = \frac{-\gamma z + 1}{z - \gamma} \tag{7}$$

where $U_\alpha(z)$ and $U_\beta(z)$ are the z-transform of the signals u_α (filter input) and u_β (filter output), respectively and

$$\gamma = \frac{-\sin(n\omega t_s) + 1}{\cos(n\omega t_s)} \tag{8}$$

Once u_β is obtained, the signal $\boldsymbol{u}_{\alpha\beta}$ can be treated as a vector, and then, a rotation matrix $\mathbf{R}(n\theta)$ can be used to transform $\boldsymbol{u}_{\alpha\beta}$:

$$\begin{bmatrix} u_\alpha^n \\ u_\beta^n \end{bmatrix} = \underbrace{\begin{bmatrix} \cos(n\theta) & \sin(n\theta) \\ -\sin(n\theta) & \cos(n\theta) \end{bmatrix}}_{\mathbf{R}(n\theta)} \begin{bmatrix} u_\alpha \\ u_\beta \end{bmatrix} \tag{9}$$

Once rotated, a LPF can be used to extract the constant value of $\boldsymbol{u}_{\alpha\beta}^n$ as shown in Fig. 2.

So far, the use of SRF's for harmonic extraction seems to be very costly due to the large number of trigonometric functions to be evaluated.

IV. Recursive computation of SRFs

A. Moivre's theorem application

Moivre's theorem establishes that, for any integer number k:

$$(\cos\theta + j\sin\theta)^k = \left(e^{j\theta}\right)^k = \cos(k\theta) + j\sin(k\theta) = e^{jk\theta} \tag{10}$$

Therefore, to calculate $\cos(k\theta)$ and $\sin(k\theta)$, which are necessary in multiple SRF applications, it is enough to compute $(\cos\theta + j\sin\theta)^k$. This calculation can be easily carried out in a recursive way with a single sin/cos computation ($\cos\theta$, $\sin\theta$) which are necessary for Park's transform into a reference frame synchronously rotating with the positive sequence of the fundamental frequency:

$$e^{j(k+1)\theta} = (\cos\theta + j\sin\theta)^k (\cos\theta + j\sin\theta) \tag{11}$$

and in matrix form:

$$\begin{bmatrix} \cos((k+1)\theta) \\ \sin((k+1)\theta) \end{bmatrix} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix} \begin{bmatrix} \cos(k\theta) \\ \sin(k\theta) \end{bmatrix} \tag{12}$$

Furthermore,

$$(\cos\theta + j\sin\theta)^{-k} = \cos(k\theta) - j\sin(k\theta) \tag{13}$$

which is the conjugate number of (10). Generalising (11):

$$e^{j(m+h)\theta} = e^{jm\theta}e^{jh\theta} \tag{14}$$

and, in matrix form:

$$\begin{bmatrix} \cos((m+h)\theta) \\ \sin((m+h)\theta) \end{bmatrix} = \begin{bmatrix} \cos(m\theta) & -\sin(m\theta) \\ \sin(m\theta) & \cos(m\theta) \end{bmatrix} \begin{bmatrix} \cos(h\theta) \\ \sin(h\theta) \end{bmatrix} \tag{15}$$

V. Recursive transformation of signals

When referring any electrical magnitude (voltage or current) to a SRF for harmonic analysis, the work done for lower-order harmonics can be fully utilized. For example, let's consider harmonic $(m+h)^{th}$ of sequence i:

$$\boldsymbol{u}_{dq}^{m+h-i} = e^{-i\cdot j(m+h-i)\theta}\boldsymbol{u}_{dq} \tag{16}$$

which can be rewritten as:

$$\boldsymbol{u}_{dq}^{m+h-i} = e^{-i\cdot jm\theta} \underbrace{\left(e^{-i\cdot j(h-i)\theta}\boldsymbol{u}_{dq}\right)}_{\boldsymbol{u}_{dq}^{h-i}} \tag{17}$$

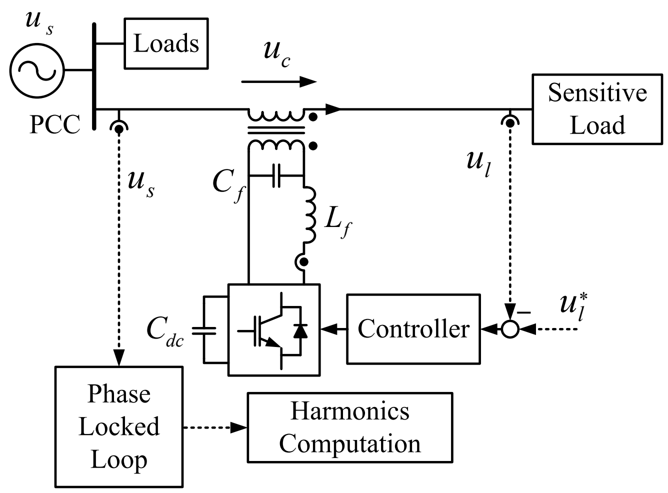

Fig. 3. A Series Active Power Filter (SeAPF) to compensate voltage harmonic distortion

or in matrix form:

$$\begin{bmatrix} u_d^{m+h-i} \\ u_q^{m+h-i} \end{bmatrix} = \begin{bmatrix} \cos(m\theta) & i \cdot \sin(m\theta) \\ -i \cdot \sin(m\theta) & \cos(m\theta) \end{bmatrix} \begin{bmatrix} u_d^{h-i} \\ u_q^{h-i} \end{bmatrix} \quad (18)$$

This, together with the discussion in Section IV, can be used to minimize the computational burden when referring a signal to a higher-order SRF, above all when not all harmonics are required for analysis. Note that, each time (18) is applied, only 4 multiplications and 2 sums are required.

VI. Application example

The algorithm proposed in this paper will be tested on a power electronics application to show its viability. The application consist on a Series Active Power Filter (SeAPF), that is aimed to protect a sensitive load when the grid voltage (u_s) is polluted with harmonics (see Fig 3). The SeAPF consist on an electronic Voltage Source Converter (VSC), a constant DC-link voltage, an AC filter and a coupling transformer which is series connected with the line. The SeAPF injects the required voltage (u_c) so that voltage imbalance and voltage harmonics are compensated at the point of connection of the sensitive load. The injected voltage is closed-loop controlled using a DSP. This controller is implemented using a dq reference frame synchronised with the positive sequence of the voltage at the PCC, using a PLL. The so called repetitive-controller is used to eliminate harmonic distortion at the load side. The goal of the experiment is to measure the harmonic contents at both grid and load voltage to evaluate the performance of the SeAPF.

A. Prototype description

The system nominal RMS voltage is 230 V and 50 Hz. The SeAPF consists of a 2-level 3-leg IGBT-based VSC with 400 V DC voltage and a 6-kVA coupling transformer with unity turn ratio and Ynz11 connection. The Y side of the transformer is connected to the VSC, and the neutral wire is not connected. This transformer has a leakage inductance of $L = 3$ mH. The filter capacitor value is $C_f = 20\ \mu F$ and the filter inductance is $L_f = 1.5$ mH, giving a 900 Hz cut-off frequency for the $L_f + C_f$ filter. The inverter switching frequency and the sampling frequency have been made equal to 5.4 kHz. To implement the controller in real time, *dSpace* and *Control Desk* are used. The grid voltage is generated using a 12 kVA Pacific-AMX programmable voltage source.

B. Harmonic extraction method for three-phase signals

The Pacific-AMX programmable voltage source has been used to simulate a grid of ω fundamental frequency and balanced harmonics of frequency $n = (6k \pm 1)\omega$ up to the 31^{st}. Harmonics $n = 5, 11, 17, 23, 29$ will be of negative sequence whilst harmonics $n = 7, 13, 19, 25, 31$ will be of positive sequence. The fastest way to compute the transformation for negative-sequence harmonics is to use (18) recursively with $m = 6$, $i = -1$ and $h = 1, 5, 11, 17, 23, 29$:

$$\begin{bmatrix} u_d^{h+6-i} \\ u_q^{h+6-i} \end{bmatrix} = \begin{bmatrix} \cos(6\theta) & i \cdot \sin(6\theta) \\ -i \cdot \sin(6\theta) & \cos(6\theta) \end{bmatrix} \begin{bmatrix} u_d^{h} \\ u_q^{h} \end{bmatrix} \quad (19)$$

For positive sequence harmonics (18) can also be used, $m = 6$, $i = 1$ and $h = 1, 7, 13, 19, 25, 31$.

The value of $\cos(6\theta)$ and $\sin(6\theta)$ is obtained first computing $\cos(2\theta)$ and $\sin(2\theta)$ ($h = 1$ and $m = 1$ in (15)), as follows:

$$\begin{bmatrix} \cos(2\theta) \\ \sin(2\theta) \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix} \quad (20)$$

Then 3θ ($h = 1$ and $m = 2$ in (15)), and finally, 6θ ($h = 3$ and $m = 3$ in (15)). The total number of multiplications (M) and sums (S) performed to obtain the harmonics are:

- Computation of reference frames rotating at $2\theta, 3\theta$, and 6θ: $3 \times (4M\ 2S)$
- Signal rotation (5 rotations at both positive and negative frequencies): $5 \times 2 \times (4M\ 2S)$
- A fifth-order Butterworth filter at each harmonic of positive and negative sequence (both real and imaginary parts): $5 \times 2 \times 2 \times (6M\ 5S)$

The number of calculations required to extract the harmonics components are summarized in Table II.

TABLE II
Number of calculations to obtain $6k \pm 1$ harmonics coefficients up to 30^{th} in DQ reference frame

OPERATION	MULT (M)	SUMS (S)	M+S
Reference Frames	12	6	18
Signal Rotations	40	20	60
Butterworth Filters	120	100	220
TOTAL	172	126	**298**

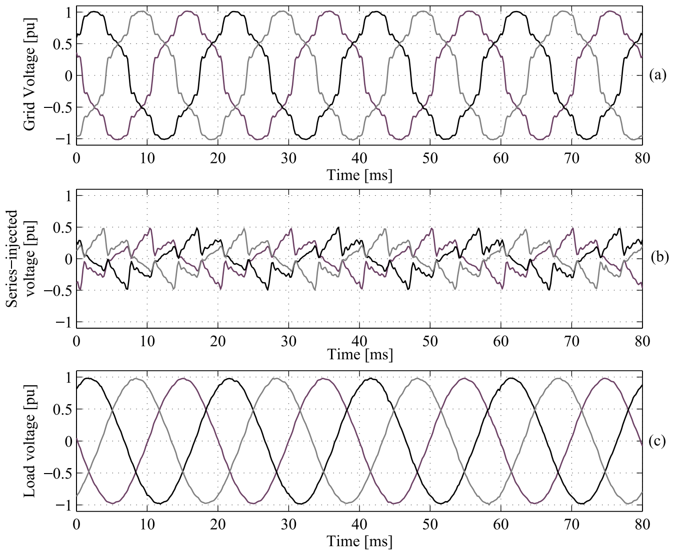

Fig. 4. Grid voltage (a), Series-injected voltage (b) and Load voltage (c) when the SAPF is filtering the grid voltage.

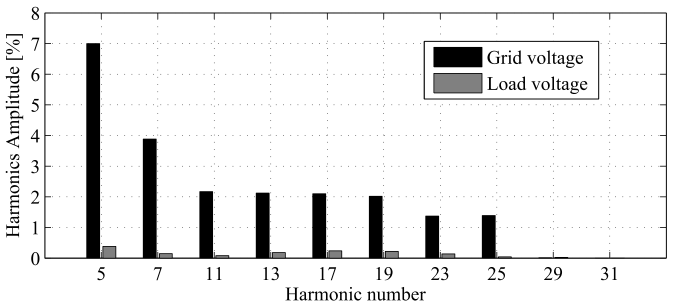

Fig. 5. Magnitude of balanced voltage harmonics on both grid and load voltage.

C. Harmonics extraction method for homopolar component

To illustrate the single-phase extraction method, the third harmonic (homopolar) from a three-phase signal will be extracted. Therefore, the following transformation is performed:

$$\begin{bmatrix} u_\alpha^3 \\ u_\beta^3 \end{bmatrix} = \begin{bmatrix} \cos(3\theta) & \sin(3\theta) \\ -\sin(3\theta) & \cos(3\theta) \end{bmatrix} \begin{bmatrix} u_\alpha \\ u_\beta \end{bmatrix} \tag{21}$$

Where u_α is the original signal and u_β is the quadrature component. To solve (21), a SRF rotating at 3θ has to be first computed. Secondly, an APF (7) tuned at 3ω is used to generate the quadrature signal of u_α. Finally, the vector components are filtered with Butterworth LPFs. The total number of calculations required are:

- Computation of reference frames rotating at 2θ, and 3θ: $2 \times (4M\ 2S)$
- Quadrature signal generation (APF): $(2M\ 2S)$
- Signal rotation (1 rotation): $(4M\ 2S)$
- A fifth-order butterworth filter at each harmonic (both real and imaginary parts): $2 \times (6M\ 5S)$

The total number of operation are 28 multiplications and 18 sums. The single-phase approach to extract harmonics is

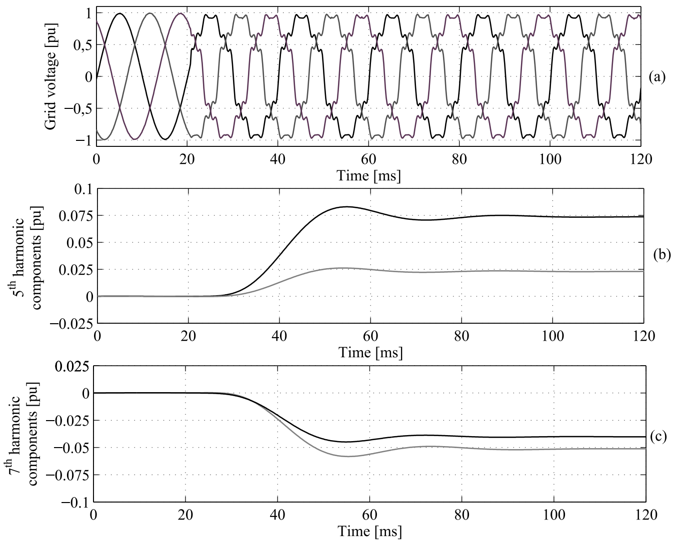

Fig. 6. Estimation of the 5^{th} and 7^{th} harmonics coefficients when the grid voltage changes. Real (black) and imaginary (grey) part.

exact when the mains has the nominal frequency. If the mains frequency varies, the phase of the APF will not be exactly -90 degrees at 3ω. If required, this problem can be solved tuning the coefficients of the APF.

VII. EXPERIMENTAL RESULTS

Fig. 4 shows the grid voltage (a), the series-injected voltage (b) and the load voltage (c). The grid voltage is balanced and polluted with harmonics and the SeAPF injects the required voltage to ameliorate the harmonic distortion at the load side. Some degree of fundamental component voltage is also injected to compensate for reactive power and to maintain the DC-link charged at its rated value. The harmonic content of the grid voltage and the load voltage is extracted using the proposed approach for three-phase signals. The steady-state magnitude of these harmonics is depicted in Fig. 5.

To evaluate the transient performance of the proposed algorithm, the voltage harmonics present in the grid are changed, rapidly. In Fig. 6 (a), the grid voltage harmonics are changed at $t = 20$ ms. The cut-off frequency of the filters has been set to 30 Hz. The real and imaginary components of the 5^{th} and 7^{th} harmonics are depicted in Fig. 6 (a) and (b), respectively. At time $t = 100$ ms, the harmonics coefficients are fully extracted. The transient behaviour is highly dependant on the filters design. The more accurate coefficient extraction, the slower the transient response is. If required, the butterworth filters could be replaced by another type of filter (eg comb filter) to obtain a faster response.

The estimation of the third harmonic from the homopolar voltage is depicted in Fig. 7. In that figure, (a) is the three-phase voltage, (b) is the homopolar voltage, and (c) are the estimated coefficients for the third harmonic. The cut-off frequency of the filters has been set, again, to 30 Hz. The grid voltage harmonics are changed at $t = 20$ ms. The performance of the estimation is similar to the three-phase case. Therefore,

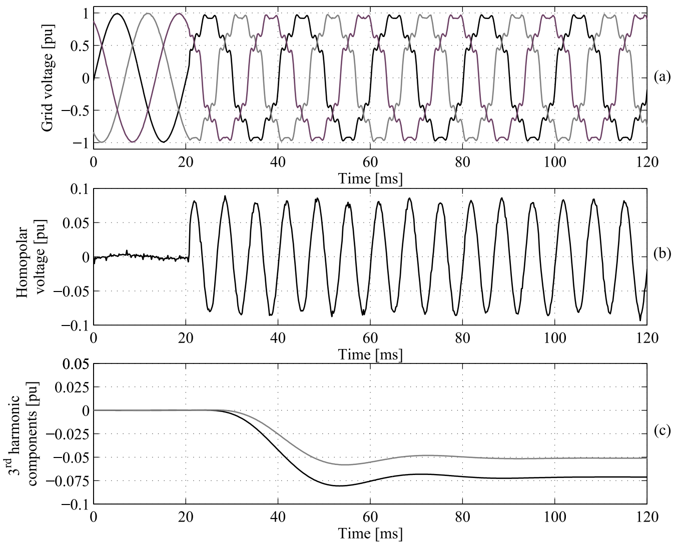

Fig. 7. Estimation of the 3^{rd} harmonic coefficients of the homopolar voltage when the grid voltage changes. Real (black) and imaginary (grey) part.

one can conclude that the use of an APF provides an easy way to handle single-phase signal as vectors.

VIII. Conclusion

Harmonic extraction methods are widely used in the industry to accurately estimate harmonic components of electrical measurements. The computational cost burden is a key point for real-time applications. This paper proposes an easy method to compute harmonics coefficients in real-time without leading a high computational cost. The algorithm is based on Multiple Reference Frames rotating synchronously with the harmonics' space vectors. Once rotated, the signals are filtered using Butterworth low-pass filters. Typically, this method is avoided due to the high number of trigonometric functions required to compute each harmonic. However, applying Moivre's theorem, trigonometric functions are avoided and replaced by multiplications and sums. This improvement has been used to minimize the number of calculations to compute harmonic coefficients. The algorithm is used to estimate the harmonic content of electrical measurements in a 5 kVA prototype of a Series Active Power Filter. The results reveal that harmonics can be accurately estimated in a reasonable time without leading high computational cost. The proposed algorithm would be very advantageous to implement controllers for Active Power Filters, which require fast computation of harmonics.

Acknowledgments

This work has been partially supported by the Spanish Government research project ENE2011-28527-C04-01.

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